SOLUTION OF CERTAIN PROBLEMS OF THE PROPAGATION OF TWO-DIMENSIONAL CONDUCTIVE JETS BY THE INTEGRAL METHOD

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The propagation of free and semibounded jets in a homogeneous magnetic field is investigated by the integral method based on the equations of a two-dimensional laminar boundary layer in the noninductive approximation ($Re_m \ll 1$) with the variation of conductivity across the jet taken approximately into account.

The application of the integral method to the solution of problems on the propagation of two-dimensional jets of conductive fluid in a homogeneous magnetic field at constant medium conductivity was described in [1]. We examine the same problems with allowance for variation of the conductivity across the jet. In particular, the injection of a strongly heated jet of conductive fluid into a low-temperature region and the converse problem are investigated. In the case of a boundary jet it is assumed that the wall is at the temperature of the surrounding medium. The variation of conductivity over the cross section of nonisothermal jets is qualitatively given as

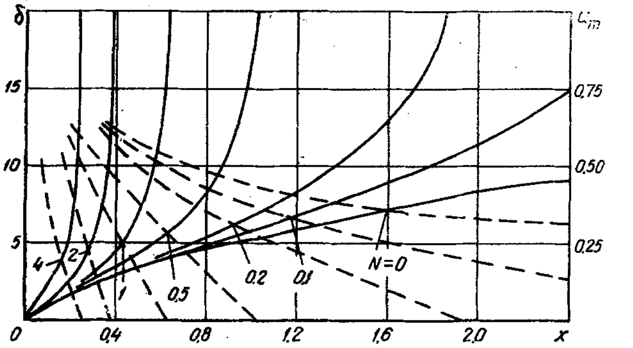

Fig. 1. Variation of u_m (dashed lines) and δ (solid lines) in the free jet at various values of N.

$$\frac{\sigma}{\sigma_0} = \frac{u}{u_m}, \quad \frac{\sigma}{\sigma_0} = 1 - \frac{u}{u_m}$$

The magnetic Reynolds numbers Re_m are assumed to be small, and the induction currents are short-circuited. A solution is presented for the "open circuit" regime and for constant medium conductivity.

Following the integral method, we look for a solution of the starting boundary layer equations in the form

$$u = u_m F(\varphi), \quad y = \delta(x)\varphi.$$

As usual, the dimensionless velocity profile is taken as a polynomial satisfying the boundary conditions. For simplicity we will limit the calculation to three terms. The functions $u_m(x)$ and $\delta(x)$ are determined from the differential equations.

We represent the boundary conditions and the expressions for the dimensionless velocity profile for a semibounded jet (a) and for a free jet (b) as follows:

$$\text{a) } F(0) = 0, \; F(\varphi_m) = 1, \; F(\infty) = 0,$$

$$\text{b) } F(0) = 1, \; F'(0) = 0, \; F(\infty) = 0,$$

$$\text{a) } F(\varphi) = \frac{27}{4}(\varphi - 2\varphi^2 + \varphi^3),$$

$$\text{b) } F(\varphi) = 1 - 2\varphi^2 + \varphi^4,$$

$$\text{a) } \varphi_m = 0.333, \quad \text{b) } \varphi_m = 0.$$

From the starting system we determine the scale functions $u_m(x)$ and $\delta(x)$ through

$$\text{a) } u_m \frac{du_m}{dx} = \nu \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=y_m} - Nu_m,$$

$$\text{b) } u_m \frac{du_m}{dx} = \nu \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} - Nu_m,$$

$$\text{a) } \frac{d}{dx}\int_0^\infty u^2 \left(\int_0^y u\,dy\right) dy = N \int_\infty^0 u \left(\int_0^y u\,dy\right) dy,$$

$$\text{b) } \frac{d}{dx}\int_{-\infty}^{\infty} u^2 dy = N \int_{\infty}^{-\infty} u\,dy,$$

which can be rewritten thus

$$\text{a) } \delta^2 \left(\frac{du_m}{dx} + N\right) + 13.5 = 0,$$

$$\text{b) } \delta^2 \left(\frac{du_m}{dx} + N\right) + 4 = 0,$$

$$\text{a) } 3\frac{du_m}{dx} + 2u\frac{d\delta}{dx} + N\frac{k_2}{k_1}\delta = 0,$$

$$\text{b) } 2\frac{du_m}{dx} + u\frac{d\delta}{dx} + N\frac{k_2}{k_1}\delta = 0.$$

Here

$$\text{a) } k_1 = \int_0^\infty F^2 \left(\int_0^\varphi F\,d\varphi\right) d\varphi = 0.011,$$

$$\text{b) } k_1 = \int_0^\infty F^2 d\varphi = 0.407,$$

$$\text{a) } k_2 = \int_0^\infty F \left(\int_0^\varphi F\,d\varphi\right) = 0.0017,$$

$$\text{b) } k_2 = \int_0^\infty F\,d\varphi = 0.533.$$

Integrating these equations for $\delta(0) = 0$, we obtain

$$\text{a) } Cx\nu = \int_0^\delta \frac{\sqrt[3]{\delta}\,d\delta}{\left(\lambda N\delta^2 + \frac{81}{4}\right)^p},$$

$$\text{b) } Cx\nu = \int_0^\delta \frac{\sqrt{\delta}\,d\delta}{(\lambda N\delta^2 + 8)^p},$$

$$\text{a) } u_m = \lambda N\frac{\delta}{\delta'} + \frac{81}{4}\frac{1}{\delta\delta'}, \quad \text{b) } u_m = \lambda N\frac{\delta}{\delta_1} + \frac{8}{\delta\delta'},$$

$$\text{a) } \lambda = \frac{3}{2} - \frac{1}{2}\frac{k_2}{k_1}, \quad p = \frac{2}{3} + \frac{1}{2\lambda}, \quad N = \frac{\sigma B^2}{\eta},$$

$$\text{b) } \lambda = 2 - \frac{k_2}{k_1}, \quad p = \frac{3}{4} + \frac{1}{2\lambda}, \quad N = \frac{\sigma B^2}{\mu}.$$

To determine the constant C we integrate the second of the starting equations along the x axis

$$\text{a) } \int_0^\infty u^2\left(\int_0^y u\,dy\right)dy + N\int_0^x\left[\int_0^\infty u\left(\int_0^y u\,dy\right)dy\right]dx =$$

$$= E_0 = \text{const};$$

$$\text{b) } \int_0^\infty u^2 dy + N\int_0^x\left(\int_0^\infty u\,dy\right)dx = I_0 = \text{const}.$$

The equations obtained are the integral "conservation conditions" for the problems in question. We note that when $N \neq 0$ both the first and second integrals in these expressions depend on x, but their sum remains constant for any value of N and equal to the value of the first integral at the point $x = 0$. As distinct from this, in the absence of a magnetic field ($N = 0$) the values of $\int_0^\infty u^2\left(\int_0^y u\,dy\right)dy$ for the semibounded jet and of $\int_0^\infty u^2 dy$ for the free jet remain constant along the jet and equal to E_0 and I_0, respectively.

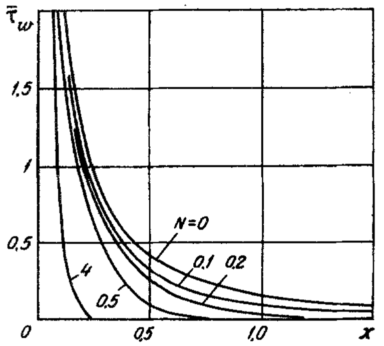

Fig. 2. Shear stress as a function of the parameter N in the semibounded jet.

Evaluating the integrals, using the solution presented above, we find that

$$\text{a) } C = \sqrt[3]{\frac{k_1}{E_0}}\left(\frac{81}{4}\right)^{1-\xi}, \quad \text{b) } C = \sqrt{\frac{k_1}{I_0}}\,(8)^{1-\xi}.$$

An analysis of the solutions obtained shows that in both cases an increase in the magnetic field leads to

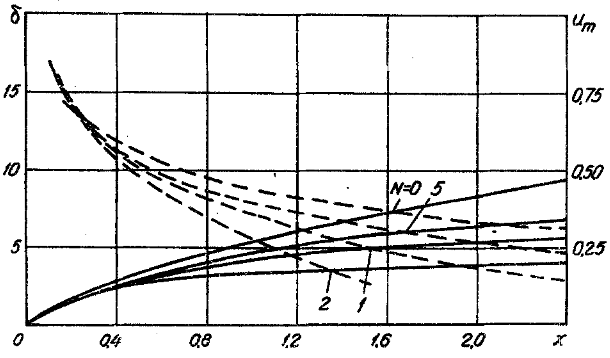

Fig. 3. Variation of u_m (dashed lines) and δ (solid lines) for the free jet.

an increase in the width of the jet, and a decrease in the maximum velocity (Fig. 1, free jet). For any value of the parameter N the jet is decelerated at a finite distance from the source ($x = 0$), as seen from Fig. 2, which shows the variation of the shear stress at the wall $\bar{\tau}_w$ $\left(\bar{\tau}_w = \frac{4}{27}\frac{\tau}{\mu\sqrt[3]{E_0}}, \tau = \mu\frac{du}{dy}\right)$ in a semibounded jet. Qualitatively, these characteristics of the effect of the magnetic field on the development of the jet also apply to the case of a strongly heated jet injected into a cold medium.

We will now consider the problem of variable conductivity. The solutions are determined from the relations presented above with the following values for the constants:

for injection of a hot jet into a cold medium

$$\text{a) } k_1 = k_2 = \int_0^1 F\left(\int_0^\varphi F^2 d\varphi\right)d\varphi = 0.369,$$

$$\text{b) } k_1 = k_2 = \int_0^\infty F^2 d\varphi = 0.416,$$

for injection of a cold jet into a hot medium

$$\text{a) } \lambda = \frac{\int_0^1 F(1-F)\,d\varphi}{\int_0^1 F^2 d\varphi} = 0.313,$$

$$\text{b) } \lambda = \frac{\int_0^1 F\left(\int_0^\varphi F(1-F)\,d\varphi\right)d\varphi}{\int_0^1 F^2\left(\int_0^\varphi F\,d\varphi\right)d\varphi} = 0.908.$$

In the latter, as distinct from the preceding cases, with an increase in the magnetic field both the effective width of the jet and the maximum velocity in a fixed section decrease (Fig. 3, free jet).

The solutions obtained relate to the short-circuit regime. If the induced currents are closed across a very large external resistance and the total current is close to zero, the integral conditions retain the same form as in the absence of a field. For example, for a hot jet injected into a cold medium the induced currents are closed within the width of the jet. In this case the solutions are determined from the corresponding expressions presented above with k_2 equal to zero ($k_2 = 0$).

The approximate expressions for the conductivity of the medium do not reflect the decrease in conductivity along the jet axis. (In this case the body force along the jet axis decreases only as a result of the fall in velocity.) This is accounted for by employing the approximation $\sigma \sim \sigma_0(u/u_*)$ ($u_* = \text{const}$). Then, the "open circuit" regime we have

$$\text{a)}\ \delta = \left(\sqrt[3]{\frac{k_1}{E_0}\,\frac{e^{2Nx}-1}{N}}\right)^{3/4},$$

$$\text{b)}\ \delta \left[\sqrt{\frac{k_1}{I_0}\,\frac{4\left(e^{3Nx}-1\right)}{N}}\right]^{2/3},$$

$$\text{a)}\ u_m = \sqrt{\frac{E_0}{\alpha}\,\frac{1}{\delta^2}};$$

$$\text{b)}\ u_m = \sqrt{\frac{I_0}{k_1}\,\frac{1}{\delta}}.$$

The difference between the solutions for the two different approximations of the conductivity equation is relatively small.

REFERENCES

1. E. V. Shcherbinin, Magnitnaya gidrodinamika [Magnetohydrodynamics], no. 3, 1965.

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